## DATA MINING - 1DL105, 1DL111

## Fall 2007

An introductory class in data mining
http://user.it.uu.se/~udbl/dut-ht2007/ alt. http://www.it.uu.se/edu/course/homepage/infoutv/ht07

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# Data in Data Mining 

(Tan, Steinbach, Kumar ch. 2)

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## What is Data?

## Attributes

- Collection of data objects and their attributes
- An attribute is a property or characteristic of an object
- Examples: eye color of a person, temperature, etc.
- Attribute is also known as variable, field, characteristic, or feature
- A collection of attributes describe an object
- Object is also known as record, point, case, sample, entity, or instance

|  | Tid | Refund | Marital Status | Taxable Income | Cheat |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Objects | 1 | Yes | Single | 125K | No |
|  | 2 | No | Married | 100K | No |
|  | 3 | No | Single | 70K | No |
|  | 4 | Yes | Married | 120K | No |
|  | 5 | No | Divorced | 95K | Yes |
|  | 6 | No | Married | 60K | No |
|  | 7 | Yes | Divorced | 220K | No |
|  | 8 | No | Single | 85K | Yes |
|  | 9 | No | Married | 75K | No |
|  | 10 | No | Single | 90K | Yes |

## Attribute Values

- Attribute values are numbers or symbols assigned to an attribute
- Distinction between attributes and attribute values
- Same attribute can be mapped to different attribute values
- Example: height can be measured in feet or meters
- Different attributes can be mapped to the same set of values
- Example: Attribute values for ID and age are integers
- But properties of attribute values can be different
- ID has no limit but age has a maximum and minimum value


## Measurement of length

- The way you measure an attribute may not match all attribute properties.



## Types of attributes

- There are different types of attributes
- Nominal
- Examples: ID numbers, eye color, zip codes
- Ordinal
- Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in \{tall, medium, short \}
- Interval
- Examples: calendar dates, temperatures in Celsius or Fahrenheit.
- Ratio
- Examples: temperature in Kelvin, length, time, counts


## Properties of attribute values

- The type of an attribute depends on which of the following properties it possesses:
- Distinctness: $=\neq$
- Order: $<>$
- Addition: + -
- Multiplication: */
- Nominal attribute: distinctness
- Ordinal attribute: distinctness \& order
- Interval attribute: distinctness, order \& addition
- Ratio attribute: all 4 properties


## Attribute types continued ...

| Attribute Type | Description | Examples | Operations |
| :---: | :---: | :---: | :---: |
| Nominal | The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. $(=, \neq)$ | zip codes, employee ID numbers, eye color, sex: \{male, female $\}$ | mode, entropy, contingency correlation, $\chi^{2}$ test |
| Ordinal | The values of an ordinal attribute provide enough information to order objects. (<, >) | hardness of minerals, \{good, better, best\}, grades, street numbers | median, percentiles, rank correlation, run tests, sign tests |
| Interval | For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. $(+,-)$ | calendar dates, temperature in Celsius or Fahrenheit | mean, standard deviation, Pearson's correlation, $t$ and $F$ tests |
| Ratio | For ratio variables, both differences and ratios are meaningful. (*, /) | temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current | geometric mean, harmonic mean, percent variation |

## Attribute types continued ...

| Attribute Level | Transformation | Comments |
| :---: | :---: | :---: |
| Nominal | Any permutation of values | If all employee ID numbers were reassigned, would it make any difference? |
| Ordinal | An order preserving change of values, i.e., new_value $=f($ old_value $)$ where $f$ is a monotonic function. | An attribute encompassing the notion of good, better best can be represented equally well by the values $\{1,2,3\}$ or by $\{0.5,1,10\}$. |
| Interval | new_value $=a *$ old_value $+b$ where a and b are constants | Thus, the Fahrenheit and Celsius temperature scales differ in terms of where their zero value is and the size of a unit (degree). |
| Ratio | new_value $=a *$ old_value | Length can be measured in meters or feet. |

## Discrete and Continuous Attributes

- Discrete Attribute
- Has only a finite or countably infinite set of values
- Examples: zip codes, counts, or the set of words in a collection of documents
- Often represented as integer variables.
- Note: binary attributes are a special case of discrete attributes
- Continuous Attribute
- Has real numbers as attribute values
- Examples: temperature, height, or weight.
- Practically, real values can only be measured and represented using a finite number of digits.
- Continuous attributes are typically represented as floating-point variables.


## Types of data sets

- Record
- Data Matrix
- Document Data
- Transaction Data
- Graph
- World Wide Web
- Molecular Structures
- Ordered
- Spatial Data
- Temporal Data
- Sequential Data
- Genetic Sequence Data


## Important Characteristics of Structured Data

- Dimensionality
- Curse of Dimensionality
- Sparsity
- Only presence counts
- Resolution
- Patterns depend on the scale


## Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

| Tid | Refund | Marital <br> Status | Taxable <br> Income |
| :--- | :--- | :--- | :--- | :--- |
| Cheat |  |  |  |$|$| 2 | No | Married | 100 K |
| :--- | :--- | :--- | :--- |
| 3 | No | Single | 70 K |
| 4 | Yes | Married | 120 K |
| 5 | No | Divorced | No |
| 6 | No | Married | 60 K |
| 7 | Yes | Divorced | 220 K |
| 8 | No | Single | 85 K |
| 9 | No | Married | Yo |
| 10 | No | Single | 90 K |

## Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multidimensional space, where each dimension represents a distinct attribute
- Such data set can be represented by an $m$ by $n$ matrix, where there are m rows, one for each object, and n columns, one for each attribute

| Projection <br> of $x$ Load | Projection <br> of $y$ load | Distance | Load | Thickness |
| :--- | :--- | :--- | :--- | :--- |
| 10.23 | 5.27 | 15.22 | 2.7 | 1.2 |
| 12.65 | 6.25 | 16.22 | 2.2 | 1.1 |

UPPSALA

## Document Data

- Each document becomes a `term' vector,
- each term is a component (attribute) of the vector,
- the value of each component is the number of times the corresponding term occurs in the document.

|  | $\begin{aligned} & \stackrel{\rightharpoonup}{\mathbb{D}} \\ & \frac{\mathrm{T}}{3} \end{aligned}$ | $\begin{aligned} & \stackrel{\mathrm{O}}{\mathbf{M}} \\ & \stackrel{1}{3} \end{aligned}$ | $=\frac{0}{0}$ | $\begin{aligned} & \text { O} \\ & \underline{\underline{0}} \end{aligned}$ | $\frac{0}{8}$ |  | 5 | $\begin{aligned} & \overline{0} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & \text { 㝘 } \\ & \text { © } \\ & \text { O } \end{aligned}$ | $\begin{aligned} & \infty \\ & \mathbb{D} \\ & \mathbb{N} \\ & \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Document 1 | 3 | 0 | 5 | 0 | 2 | 6 | 0 | 2 | 0 | 2 |
| Document 2 | 0 | 7 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 0 |
| Document 3 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 3 | 0 |

## Transaction Data

- A special type of record data, where
- each record (transaction) involves a set of items.
- For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

| TID | Items |
| :--- | :--- |
| 1 | Bread, Coke, Milk |
| 2 | Beer, Bread |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Coke, Diaper, Milk |

## Graph Data

## - Examples: Generic graph and HTML Links


<a href="papers/papers.html\#bbbb">
Data Mining </a>
<li>
<a href="papers/papers.html\#aaaa">
Graph Partitioning </a>
<li>
<a href="papers/papers.html\#aaaa">
Parallel Solution of Sparse Linear System of Equations </a>
<li>
<a href="papers/papers.html\#ffff">
N-Body Computation and Dense Linear System Solvers

## Chemical Data

- Benzene Molecule: C6H6


## Ordered Data

- Sequences of transactions

Items/Events


An element of the sequence

## Ordered Data

- Genomic sequence data

GGTTCCGCCTTCAGCCCCGCGCC CGCAGGGCCCGCCCCGCGCCGTC GAGAAGGGCCCGCCTGGCGGGCG GGGGGAGGCGGGGCCGCCCGAGC CCAACCGAGTCCGACCAGGTGCC СССТСTGCTCGGCCTAGACCTGA GCTCATTAGGCGGCAGCGGACAG GCCAAGTAGAACACGCGAAGCGC TGGGCTGCCTGCTGCGACCAGGG

## Ordered Data

- Spatio-Temporal Data

Jan

Average Monthly Temperature of land and ocean


## Data Quality

- What kinds of data quality problems?
- How can we detect problems with the data?
- What can we do about these problems?
- Examples of data quality problems:
- Noise and outliers
- missing values
- duplicate data


## Noise

- Noise refers to modification of original values
- Examples: distortion of a person's voice when talking on a poor phone and "snow" on television screen


Two Sine Waves


Two Sine Waves + Noise

## Outliers

- Outliers are data objects with characteristics that are considerably different than most of the other data objects in the data set
$\odot$



## Missing Values

- Reasons for missing values
- Information is not collected (e.g., people decline to give their age and weight)
- Attributes may not be applicable to all cases (e.g., annual income is not applicable to children)
- Handling missing values
- Eliminate Data Objects
- Estimate Missing Values
- Ignore the Missing Value During Analysis
- Replace with all possible values (weighted by their probabilities)


## Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
- Major issue when merging data from heterogeous sources
- Examples:
- Same person with multiple email addresses
- Data cleaning
- Process of dealing with duplicate data issues


## Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation


## Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)
- Purpose
- Data reduction
- Reduce the number of attributes or objects
- Change of scale
- Cities aggregated into regions, states, countries, etc
- More "stable" data
- Aggregated data tends to have less variability


## Aggregation

Variation of Precipitation in Australia


Standard Deviation of Average
Monthly Precipitation


Standard Deviation of Average
Yearly Precipitation

## Sampling

- Sampling is the main technique employed for data selection.
- It is often used for both the preliminary investigation of the data and the final data analysis.
- Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.
- Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.


## Sampling ...

- The key principle for effective sampling is the following:
- using a sample will work almost as well as using the entire data sets, if the sample is representative
- A sample is representative if it has approximately the same property (of interest) as the original set of data


## Types of Sampling

- Simple Random Sampling
- There is an equal probability of selecting any particular item
- Sampling without replacement
- As each item is selected, it is removed from the population
- Sampling with replacement
- Objects are not removed from the population as they are selected for the sample.
- In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
- Split the data into several partitions; then draw random samples from each partition


## Sample Size



## Sample Size

－What sample size is necessary to get at least one object from each of 10 groups．


## Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies
- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful

- Randomly generate 500 points
- Compute difference between max and min distance between any pair of points


## Dimensionality Reduction

- Purpose:
- Avoid curse of dimensionality
- Reduce amount of time and memory required by data mining algorithms
- Allow data to be more easily visualized
- May help to eliminate irrelevant features or reduce noise
- Techniques
- Principle Component Analysis
- Singular Value Decomposition
- Others: supervised and non-linear techniques


## Dimensionality Reduction: PCA

- Goal is to find a projection that captures the largest amount of variation in data



## Dimensionality Reduction: PCA

- Find the eigenvectors of the covariance matrix
- The eigenvectors define the new space



## Dimensionality Reduction: ISOMAP

By: Tenenbaum, de Silva, Langford (2000)

- Construct a neighbourhood graph
- For each pair of points in the graph, compute the shortest path distances - geodesic distances



## Feature Subset Selection

- Another way to reduce dimensionality of data
- Redundant features
- duplicate much or all of the information contained in one or more other attributes
- Example: purchase price of a product and the amount of sales tax paid
- Irrelevant features
- contain no information that is useful for the data mining task at hand
- Example: students' ID is often irrelevant to the task of predicting students' GPA


## Feature Subset Selection

- Techniques:
- Brute-force approch:
- Try all possible feature subsets as input to data mining algorithm
- Embedded approaches:
- Feature selection occurs naturally as part of the data mining algorithm
- Filter approaches:
- Features are selected before data mining algorithm is run
- Wrapper approaches:
- Use the data mining algorithm as a black box to find best subset of attributes


## Feature Creation

- Create new attributes that can capture the important information in a data set much more efficiently than the original attributes
- Three general methodologies:
- Feature Extraction
- domain-specific
- Mapping Data to New Space
- Feature Construction
- combining features


## Mapping Data to a New Space

- Fourier transform
- Wavelet transform


Two Sine Waves
Two Sine Waves + Noise


Frequency

## Discretization Using Class Labels

## entropy based approach



## Discretization Without Using Class Labels




Equal frequency


Equal interval width


K-means

## Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
- Simple functions: $\mathrm{x}^{\mathrm{k}}, \log (\mathrm{x}), \mathrm{e}^{\mathrm{x}},|\mathrm{x}|$
- Standardization and Normalization



## Similarity and Dissimilarity

- Similarity
- Numerical measure of how alike two data objects are.
- Is higher when objects are more alike.
- Often falls in the range $[0,1]$
- Dissimilarity
- Numerical measure of how different are two data objects
- Lower when objects are more alike
- Minimum dissimilarity is often 0
- Upper limit varies
- Proximity refers to a similarity or dissimilarity


## Similarity/Dissimilarity for Simple Attributes

- $\quad p$ and $q$ are the attribute values for two data objects.

| Attribute <br> Type | Dissimilarity | Similarity |
| :--- | :--- | :--- |
| Nominal | $d= \begin{cases}0 & \text { if } p=q \\ 1 & \text { if } p \neq q\end{cases}$ | $s= \begin{cases}1 & \text { if } p=q \\ 0 & \text { if } p \neq q\end{cases}$ |
| Ordinal | $d=\frac{\|p-q\|}{n-1}$ <br> (values mapped to integers 0 to $n-1, ~$ <br> where $n$ is the number of values) | $s=1-\frac{\|p-q\|}{n-1}$ |
| Interval or Ratio | $d=\|p-q\|$ | $s=-d, s=\frac{1}{1+d}$ or <br> $s=1-\frac{d-m i n-d}{\text { max_d-min-d }}$ |

Table 5.1. Similarity and dissimilarity for simple attributes

## Euclidean Distance

- Euclidean Distance

$$
\text { dist }=\sqrt{\sum_{k=1}^{n}\left(p_{k}-q_{k}\right)^{2}}
$$

- Where n is the number of dimensions (attributes) and pk and qk are, respectively, the kth attributes (components) or data objects p and q .
- Standardization is necessary, if scales differ.


## Euclidean Distance



| point | $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: | :---: |
| $\mathbf{p 1}$ | 0 | 2 |
| $\mathbf{p 2}$ | 2 | 0 |
| $\mathbf{p 3}$ | 3 | 1 |
| $\mathbf{n 4}$ | 5 | 1 |


|  | p1 | p2 | p3 | p4 |
| :---: | ---: | ---: | ---: | ---: |
| $\mathbf{p 1}$ | 0 | 2.828 | 3.162 | 5.099 |
| $\mathbf{p 2}$ | 2.828 | 0 | 1.414 | 3.162 |
| $\mathbf{p 3}$ | 3.162 | 1.414 | 0 | 2 |
| $\mathbf{n 4}$ | 5.099 | 3.162 | 2 | 0 |

Distance Matrix

## Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$
\operatorname{dist}=\left(\sum_{k=1}^{n}\left|p_{k}-\boldsymbol{q}_{\boldsymbol{k}}\right|^{\boldsymbol{r}}\right)^{\frac{1}{r}}
$$

- Where r is a parameter, n is the number of dimensions (attributes) and pk and qk are, respectively, the kth attributes (components) or data objects p and $q$.


## Minkowski Distance: examples

- $\mathrm{r}=1$. City block (Manhattan, taxicab, L1 norm) distance.
- A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r=2$. Euclidean distance
- $\mathrm{r} \rightarrow \infty$. "supremum" (Lmax norm, $\mathrm{L} \infty$ norm) distance.
- This is the maximum difference between any component of the vectors
- Do not confuse r with n , i.e., all these distances are defined for all numbers of dimensions.


## Minkowski Distance

| point | $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: | :---: |
| $\mathbf{p 1}$ | 0 | 2 |
| $\mathbf{p 2}$ | 2 | 0 |
| $\mathbf{p 3}$ | 3 | 1 |
| $\mathbf{n 4}$ | 5 | 1 |


| $\mathbf{L 1}$ | $\mathbf{p 1}$ | $\mathbf{p 2}$ | $\mathbf{p 3}$ | $\mathbf{p 4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{p 1}$ | 0 | 4 | 4 | 6 |
| $\mathbf{p 2}$ | 4 | 0 | 2 | 4 |
| $\mathbf{p 3}$ | 4 | 2 | 0 | 2 |
| $\mathbf{n 4}$ | 6 | 4 | 2 | 0 |


| $\mathbf{L 2}$ | $\mathbf{p 1}$ | $\mathbf{p 2}$ | $\mathbf{p 3}$ | $\mathbf{p 4}$ |
| ---: | ---: | ---: | ---: | ---: |
| $\mathbf{p 1}$ | 0 | 2.828 | 3.162 | 5.099 |
| $\mathbf{p 2}$ | 2.828 | 0 | 1.414 | 3.162 |
| $\mathbf{p 3}$ | 3.162 | 1.414 | 0 | 2 |
| $\mathbf{n 4}$ | 5.099 | 3.162 | 2 | 0 |


| $\mathbf{L} \infty$ | $\mathbf{p 1}$ | $\mathbf{p 2}$ | $\mathbf{p 3}$ | $\mathbf{p 4}$ |
| :--- | ---: | ---: | ---: | ---: |
| $\mathbf{p 1}$ | 0 | 2 | 3 | 5 |
| $\mathbf{p 2}$ | 2 | 0 | 1 | 3 |
| $\mathbf{p 3}$ | 3 | 1 | 0 | 2 |
| $\mathbf{n 4}$ | 5 | 3 | 2 | 0 |

Distance Matrix

## Mahalanobis Distance

$$
\text { mahalanobis }(p, q)=(p-q) \Sigma^{-1}(p-q)^{T}
$$



For red points, the Euclidean distance is 14.7, Mahalanobis distance is 6.
-

## Mahalanobis Distance



Covariance Matrix:

$$
\Sigma=\left[\begin{array}{ll}
0.3 & 0.2 \\
0.2 & 0.3
\end{array}\right]
$$

A: $(0.5,0.5)$
B: $(0,1)$
C: $(1.5,1.5)$

Mahal $(A, B)=5$
Mahal(A,C) $=4$

## Common properties of a distance

- Distances, such as the Euclidean distance, have some well known properties.
- $d(p, q) \geq 0$ for all $p$ and $q$ and $d(p, q)=0$ only if $p=q$. (Positive definiteness)
$-\mathrm{d}(\mathrm{p}, \mathrm{q})=\mathrm{d}(\mathrm{q}, \mathrm{p})$ for all p and q . (Symmetry)
$-\mathrm{d}(\mathrm{p}, \mathrm{r}) \leq \mathrm{d}(\mathrm{p}, \mathrm{q})+\mathrm{d}(\mathrm{q}, \mathrm{r})$ for all points $\mathrm{p}, \mathrm{q}$, and r . (Triangle Inequality)
where $\mathrm{d}(\mathrm{p}, \mathrm{q})$ is the distance (dissimilarity) between points (data objects), p and q.
- A distance that satisfies these properties is a metric


## Common properties of a similarity

- Similarities, also have some well known properties.
$-\mathrm{s}(\mathrm{p}, \mathrm{q})=1$ (or maximum similarity) only if $\mathrm{p}=\mathrm{q}$.
$-s(p, q)=s(q, p)$ for all $p$ and $q$. (Symmetry)
where $s(p, q)$ is the similarity between points (data objects), $p$ and q.


## Similarity between binary vectors

- Common situation is that objects, p and q , have only binary attributes
- Compute similarities using the following quantities

M01 $=$ the number of attributes where p was 0 and q was 1
$\mathrm{M} 10=$ the number of attributes where p was 1 and q was 0
M00 $=$ the number of attributes where p was 0 and q was 0
M11 $=$ the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

SMC $=$ number of matches $/$ number of attributes

$$
=(\mathrm{M} 11+\mathrm{M} 00) /(\mathrm{M} 01+\mathrm{M} 10+\mathrm{M} 11+\mathrm{M} 00)
$$

$\mathrm{J}=$ number of 11 matches $/$ number of not-both-zero attributes values

$$
=(\mathrm{M} 11) /(\mathrm{M} 01+\mathrm{M} 10+\mathrm{M} 11)
$$

## SMC versus Jaccard: example

- p = 1000000000
- q = 0000001001
- $\mathrm{M} 01=2$ (the number of attributes where p was 0 and q was 1 )
- M10 $=1 \quad$ (the number of attributes where p was 1 and q was 0 )
- M00 $=7$ (the number of attributes where p was 0 and q was 0 )
- M11 $=0$ (the number of attributes where p was 1 and q was 1 )
- $\quad \mathrm{SMC}=(\mathrm{M} 11+\mathrm{M} 00) /(\mathrm{M} 01+\mathrm{M} 10+\mathrm{M} 11+\mathrm{M} 00)=(0+7) /(2+1+0+7)=$ 0.7
- J $=(\mathrm{M} 11) /(\mathrm{M} 01+\mathrm{M} 10+\mathrm{M} 11)=0 /(2+1+0)=0$


## Cosine Similarity

- If d1 and d2 are two document vectors, then

$$
\cos (\mathrm{d} 1, \mathrm{~d} 2)=(\mathrm{d} 1 \cdot \mathrm{~d} 2) /\|\mathrm{d} 1\|\|\mathrm{d} 2\|,
$$

where • indicates vector dot product and $\|\mathrm{d}\|$ is the length of vector d .

- Example:

$$
\begin{aligned}
& \mathrm{d} 1=3205000200 \\
& \mathrm{~d} 2=1000000102 \\
& \mathrm{~d} 1 \cdot \mathrm{~d} 2=3 * 1+2 * 0+0 * 0+5 * 0+0 * 0+0 * 0+0 * 0+2 * 1+0 * 0+0 * 2=5 \\
& \|\mathrm{~d} 1\|=(3 * 3+2 * 2+0 * 0+5 * 5+0 * 0+0 * 0+0 * 0+2 * 2+0 * 0+0 * 0)^{0.5}=(42)^{0.5}=6.481 \\
& \|\mathrm{~d} 2\|=(1 * 1+0 * 0+0 * 0+0 * 0+0 * 0+0 * 0+0 * 0+1 * 1+0 * 0+2 * 2)^{0.5}=(6)^{0.5}=2.245 \\
& \cos (\mathrm{~d} 1, \mathrm{~d} 2)=.3150
\end{aligned}
$$

## Correlation

- Correlation examines the degree to which the values for two variables behave similarly by measuring the linear relationship between objects.

$$
c(x, y)=\frac{\sum\left(x_{i}-\bar{X}\right)\left(y_{i}-\bar{Y}\right)}{\sqrt{\sum\left(x_{i}-\bar{X}\right)^{2} \sum\left(y_{i}-\bar{Y}\right)^{2}}}
$$

- Correlation coefficient:
- 1 = perfect positive correlation
- $-1=$ perfect negative correlation
- $0=$ no correlation
- Correlation $=-1: \quad \mathrm{X}=(-3,6,0,3,-6)$

$$
\mathrm{Y}=(1,-2,0,-1,2)
$$

- Correlation $=+1: \quad X=(3,6,0,3,6)$

$$
\mathrm{Y}=(1,2,0,1,2)
$$

## Visually Evaluating Correlation



Scatter plots showing the similarity from -1 to 1.

0.50



## General Approach for Combining Similarities

- Sometimes attributes are of many different types, but an overall similarity is needed.

1. For the $k^{t h}$ attribute, compute a similarity, $s_{k}$, in the range $[0,1]$.
2. Define an indicator variable, $\delta_{k}$, for the $k_{t h}$ attribute as follows:
$\delta_{k}= \begin{cases}0 & \text { if the } k^{t h} \text { attribute is a binary asymmetric attribute and both objects have } \\ \text { a value of } 0, \text { or if one of the objects has a missing values for the } k^{t h} \text { attribute } \\ 1 & \text { otherwise }\end{cases}$
3. Compute the overall similarity between the two objects using the following formula:

$$
\operatorname{similarity}(p, q)=\frac{\sum_{k=1}^{n} \delta_{k} s_{k}}{\sum_{k=1}^{n} \delta_{k}}
$$

## Using weights to combine similarities

- May not want to treat all attributes the same.
- Use weights $\mathrm{w}_{\mathrm{k}}$ which are between 0 and 1 and sum to 1 .

$$
\begin{aligned}
& \operatorname{similarity}(p, q)=\frac{\sum_{k=1}^{n} w_{k} \delta_{k} s_{k}}{\sum_{k=1}^{n} \delta_{k}} \\
& \operatorname{distance}(p, q)=\left(\sum_{k=1}^{n} w_{k}\left|p_{k}-q_{k}\right|^{r}\right)^{1 / r}
\end{aligned}
$$

## Density

- Density-based clustering require a notion of density
- Examples:
- Euclidean density
- Euclidean density = number of points per unit volume
- Probability density
- Graph-based density


## Euclidean density: cell-based

- Simplest approach is to divide region into a number of rectangular cells of equal volume and define density as \# of points the cell contains


Figure 7.13. Cell-based density.

| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 17 | 18 | 6 | 0 | 0 | 0 |
| 14 | 14 | 13 | 13 | 0 | 18 | 27 |
| 11 | 18 | 10 | 21 | 0 | 24 | 31 |
| 3 | 20 | 14 | 4 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 7.6. Point counts for each grid cell.

## Euclidean Density - Center-based

- Euclidean density is the number of points within a specified radius of the point


Figure 7.14. Illustration of center-based density.

